DESIGN AND IMPLEMETATION OF NETWORK LOCALIZATION SERVICE USING ANGLE-INDEXED SIGNAL STRENGTH MEASUREMENTS

An Honor Thesis

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ABSTRACT

The recent technological developments in low-power electronics and wireless communication have increased the use of ad hoc wireless sensor networks for environmental monitoring and security applications. A typical network is a randomly distributed collection of scores of low-power wireless sensor nodes. Network localization is the process of estimating the location of each sensor in the network. Network localization is an imperative capability for the effective operation of wireless sensor networks. In this research project we explore an innovative approach to localization that uses an array of directional antennas, together with a node's radio communications, to sense the bearing and distance between neighboring nodes. We also pursue a variety of computational methods to determine the best position estimator.

We found that the proposed localization approach can measure the angle of arrival (AOA) with about 4 degrees mean absolute error and can determine distance between the transmitter and receiver with 5 feet mean absolute error using the prototype antenna array. In addition, the mean absolute distance error between the actual and estimated positions was found to be 14.5 feet (about 4.5 meters). Also, compared to other approaches, our approach can solve the network localization problem with fewer beacon nodes needed in the network, which means a significant reduction in hardware and software cost.

Our findings demonstrate the feasibility of non-coherently using directional arrays in wireless sensor networks for determining angle from radio signal strength. The estimated angles, combined with either coarse distance information or a few known node locations, provide a solution for the network localization task.

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1. Introduction

In recent years, there has been an increase in the use of ad hoc wireless sensor networks for environmental monitoring applications and military surveillance. In a typical wireless sensor network, there is a random distribution of wireless sensor nodes. These sensor nodes are typically capable of operating with minimum user attendance and minimum power consumption. Sensor networks usually consist of a large number of sensor nodes that communicate with each other. Sensor measurements at a node are useful only if the sensor location is known. Therefore, knowing node position becomes important.

Network localization, i.e. the process of estimating the location of each sensor in the network, is an essential step for effective functioning of large sensor networks. In fact, the network localization problem is an unavoidable challenge, and determining manually node location in a network is not practical, especially if the sensor network consists of a large number of densely deployed sensor nodes. Another possible method is global positioning system (GPS) [1]. But GPS is expensive in terms of hardware cost and power consumption for large sensor networks. Also, recent research work (e.g. [2], [3] and [4]) in network localization has proposed to estimate the distance between two sensor nodes using the radio received signal strength (RSS). But experimental results have shown that this approach provides inaccurate results due to variable propagation losses.

In this research project, our main goal is to develop a technique that accurately estimates sensor locations in large networks of low-power wireless sensor nodes. We exploit radio frequency (RF) communications signals to sense RSS. We use a non-coherent array of directional antennas to collect RSS measurements. The directional antennas we are using concentrate their communication capacity in one direction and provide magnitude increase in communication range by a factor of two. We hypothesize that the signal strengths measured by the array of directional antennas may be used to accurately estimate the angle or bearing between the transmitting and receiving nodes. We also hypothesize that the RSS measurements collected by the array of directional antennas can be used to determine more accurate estimation of the distance between the transmitter and receiver nodes.

To investigate our hypotheses, we used commercial radios and a custom antenna array to conduct field measurements of received signal strengths at a two-antenna array, as a function of both distance and angle to the transmitter. The measured RSS values are modeled as

$$RSS = f(d, \theta) \tag{1}$$

where d and θ are the distance and bearing between the receiver and transmitter nodes, respectively. Then, we implemented a variety of computational algorithms of distance and angle from observed signal strengths based on conditional means or modes.

In this report, we start with a brief explanation of the concept of bearing between two nodes. Then, we discuss several methods that can be pursued to solve the network localization by estimating the angle of arrival (AOA). We also derive a formula to compute the posterior probability $P(d, \theta \mid RSS)$ from the collected RSS measurements. Then, we present the results from the different techniques that we pursued to estimate the distance and bearing between the transmitter and receiver. Finally, we obtain the position estimation from the computed distance and bearing estimates.

2. The Concept of Bearing and AOA

The concept of bearing is discussed in literature [e.g. [5]], but to prevent any confusion, it is important to define our concept of bearing. First, we assume that each node in a network has an axis that defines its orientation. During all our field-testing, the transmitter's axis was always pointing toward the receiver, and the bearing angle between the receiver and transmitter is the angle θ as shown in Figure 2.1.

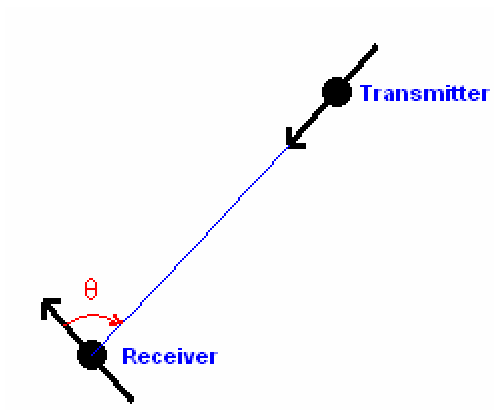


Figure 2.1: The bearing between the transmitter and receiver nodes (receiver has a single antenna)

The receiver node in our case has two-antenna array, and the acute angle between the two antennas (Beam (A) and Beam (B)) is 60^o. We define the axis of the receiver to be pointing in the direction of the bisector of the acute angle as shown in Figure 2.2. The bearing angle θ increases positively toward beam (A) and increases negatively toward beam (B).

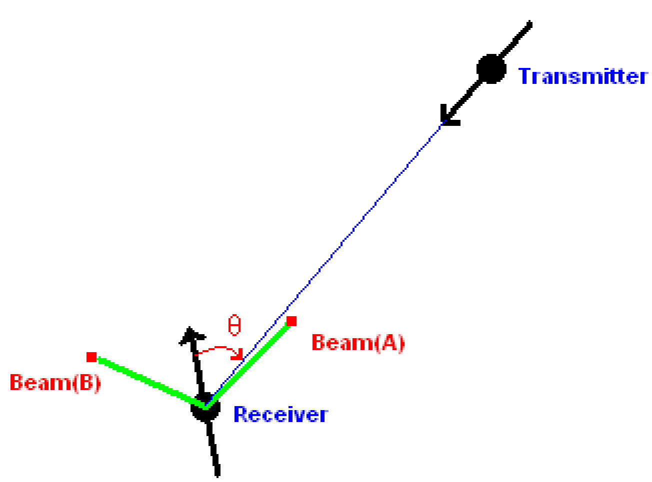


Figure 2.2: The bearing between the transmitter and receiver nodes (receiver has two antennas)

Finally, throughout this report the bearing angle θ between the transmitter and receiver will be given the name *angle of arrival* (AOA).

3. Estimating AOA to Solve the Network Localization Problem

Most recent research work in network localization has presented techniques that utilize the distance information between neighboring nodes to estimate positions in a network. Using the distance information only, it is possible to determine the location of a node uniquely (one solution) in a plane if the node has three or more neighboring beacons [6], where a beacon is a node with known location. This technique is not practical because, in some special cases (for example, nodes that are located near the borders of the network), a node's communication is limited to two or one beacon node. Therefore, using the distance information to localize in these special cases results in inaccurate and uncertain position estimation. Also, as it is shown in [7], using the distance information requires a large density of beacon nodes in the network for good localization. In this paper, we propose to utilize the AOA sensing capability of a node to reduce the number of beacon nodes needed to uniquely localize.

There are two possible methods that we identified that utilize the AOA sensing capability of a beacon node to solve for an unknown node location. The first method is called Two Beacons with AOA (TBwA); two beacons sense the AOA from the unknown transmitter to estimate the location of this unknown node. The second method is called One Beacon with AOA and Distance Information (OBADI). For this method, a beacon estimates the AOA and distance with respect to the transmitter node from the observed RSS. Now, we describe each method in the noiseless case in more details.

3.1 Two Beacons with AOA (TBwA)

In this method, we utilize the AOA sensing capability of two beacon nodes to locate a third node as shown in Figure 3.1. Angles A and C are determined using the AOA sensing capabilities of the two beacons; b can be easily computed since the positions of the beacons are known.

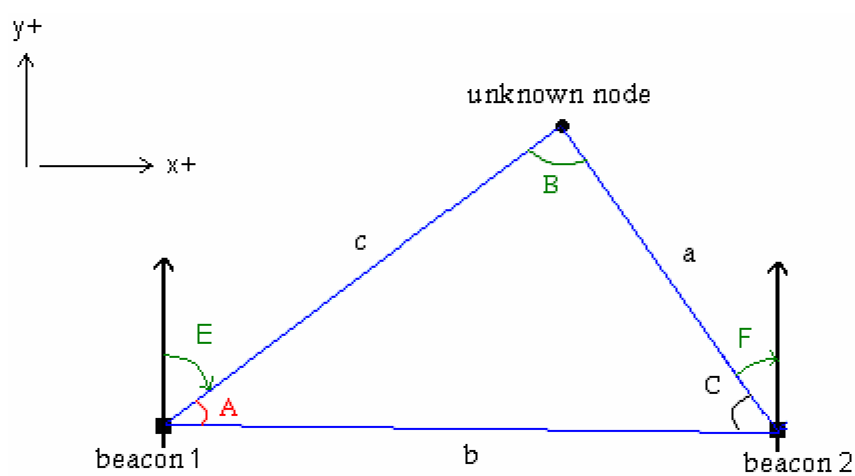


Figure 3.1: Visual illustration of the TBwA method.

Given all this, B can be computed using the 180^o rule of a triangle:

$$B = 180^o - C - A \tag{2}$$

and, c and a can be computed using the two equations of the Law of Sines:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \tag{3}$$

Assuming the beacons' axis and y-axis are parallel, and (x_1, y_1) and (x_2, y_2) are the positions of beacons (1) and (2), respectively, we notice that the angle of arrivals E and F are positive and negative, respectively. Hence, the position of the unknown node $(x \;, y \;)$:

$$x = x_1 + (c)\sin(E) = x_2 + (a)\sin(F)$$
$$y = y_1 + (c)\cos(E) = y_2 + (a)\cos(F) \tag{4}$$

3.2 One Beacon with AOA and Distance Information (OBADI)

As shown in Figure 3.2, we utilize the AOA sensing capability of the beacon and determine the distance information from the observed RSS to compute the polar coordinates (r, θ) of the unknown node with respect to the beacon.

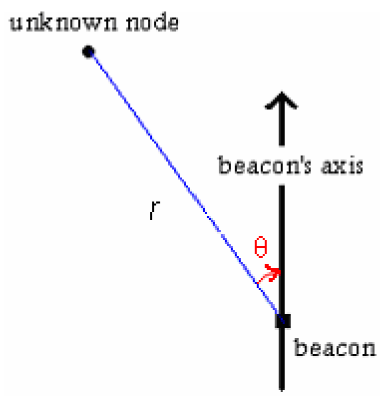


Figure 3.2: Visual illustration of the OBADI method.

Let (x_b, y_b) be the known position of the beacon. Hence, the position of the unknown node $(x \;, y \;)$:

$$x = x_b + (r)\sin(\theta)$$
$$y = y_b + (r)\cos(\theta) \tag{5}$$

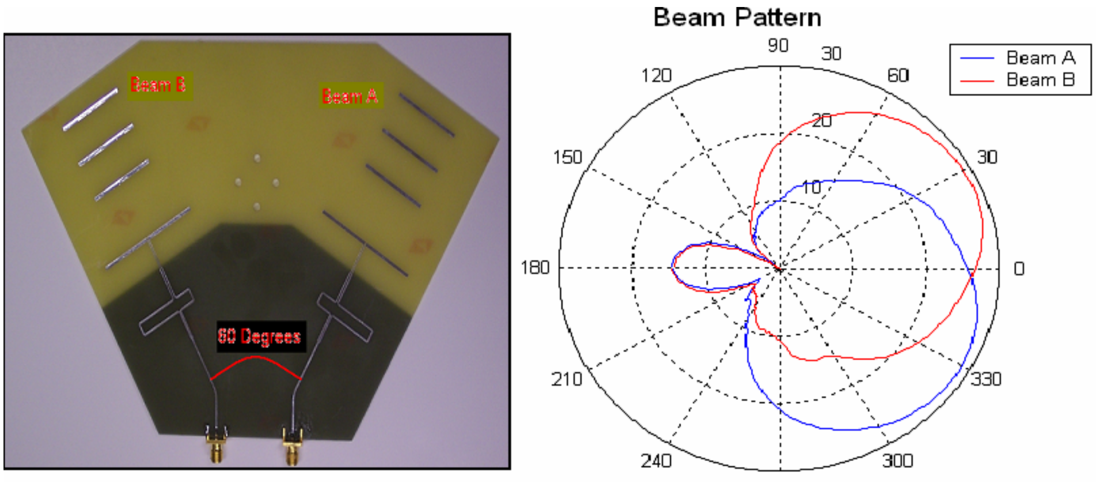
3.3 Comparison and Analysis

Each of these two methods has strengths and weaknesses. The TBwA requires a relatively more complex computation procedure compared to the OBADI method. Also, in the TBwA method, a communication link must exist between the two beacons to exchange the observed information to locate the unknown node. The OBADI method, on the other hand, is more practical because one neighboring beacon is needed to locate the unknown node; therefore, fewer beacons are needed in the network. Hence, implementing the OBADI method to localize a single node is less complex in terms of computation and less expensive in terms of hardware and software costs. In this paper, we implement the OBADI method technique to estimate the polar coordinates (r,θ) of the transmitter with respect to the beacon receiver.

4. Experimental Procedure and Data Collection and Processing

4.1 Experimental Procedure

Field-testing was performed on a grassy football field west of The Ohio State University campus to collect a wide range of measurements. Stargate processor board using SMC 802.11b communication card recorded the strength measurements of the received radio frequency signal [8, 9]. The two-directional antenna array receiver is shown in Figure 4.1a. Each antenna is a Quasi-Yagi design; OSU graduate student Min-Young Nam designed the antenna in Ansoft HFSS. A 60^{o} angle separates the two beams and there is a 60^{o} angle separation between the maximum radiation gains of beams (A) and (B) as shown in Figure 4.1b. The beam pattern was measured by OSU graduate student Josh Ash at the Electroscience Laboratory in the Ohio State University campus. The transmitter antenna's power and the receiver antenna's power were kept constant throughout this process of measurement accumulation. In addition, the transmitter and receiver antennas' height from the ground was kept constant at about one meter throughout the experiment. In Figure 4.2, two pictures were taken during field-test that show the experiment set up.



(a) (b)

Figure 4.1: (a) Quasi-yagi antenna array, (b) Beam pattern of the antenna array.

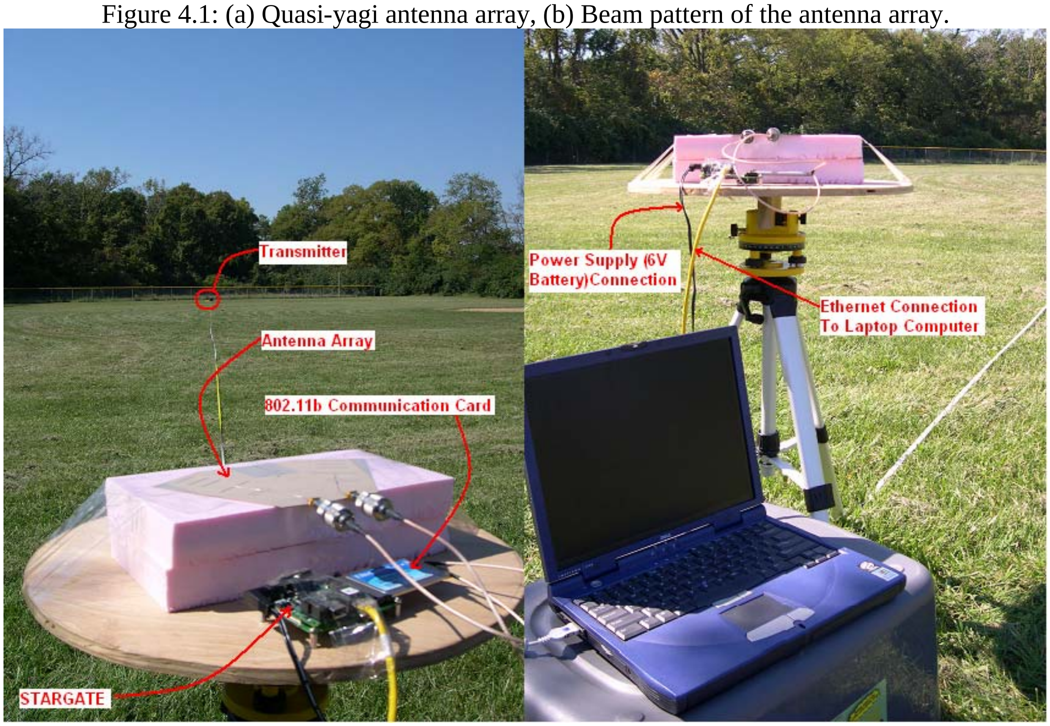


Figure 4.2: Two pictures taken during field-test.

4.2 Data Collection

The set of collected measurements contains the RSS observations recorded by the two-antenna array receiver as the transmitter was positioned on the dots shown in Figure 4.3. Each packet contains the received signal strength observed by beam (A) RSSA, the received signal strength observed by beam (B) RSSB, the polar coordinates (r,θ) of the transmitter position with respect to the receiver and noise observed by the two beams. At each position, both beams (A) and (B) observed 400 packets each. The range of RSSA is 69-87 and the range of RSSB is 74-88.

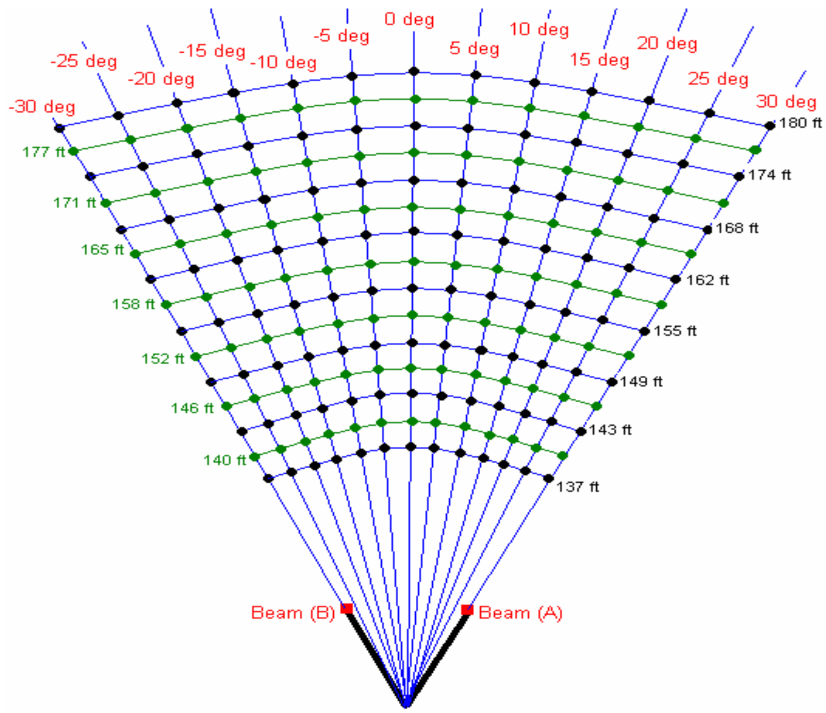


Figure 4.3: The positions where the transmitter antenna was placed

4.3 Data Processing

From the raw data, we extracted 372 distance and angle distributions; one distance distribution and one angle distribution for each of the 186 $RSS = \{RSSA, RSSB\}$ pairs observed during field-testing. In addition, nine distance distributions were extracted; one distance distribution for each $RSSA$, ranging from 79 to 87, observed when beam (A) was pointing directly at the transmitter. Similarly, ten distance distributions were extracted for the ten $RSSB$ values, ranging from 79 to 88.

Asymmetrical characteristics have been observed in the normal representations of the AOA distributions as shown in Figure 4.4. At RSSA = RSSB, it is expected that the mean AOA is 0^o for identical beams; however, a positive offset of approximately 5^o is noted. This is consistent with the behavior of radiation gains of both beams shown in Figure 4.1b, which also intersect at around 5^{o}. This shows that beam (B) has relatively more powerful sensing capabilities than beam (A). However, this should not affect our localization results because we use the same two beams for both building the database (signal map) and RSS detection.

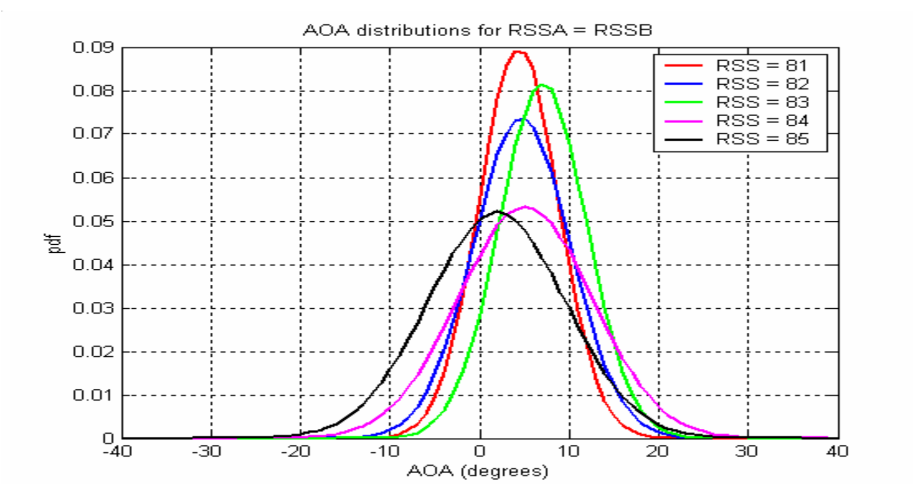


Figure 4.4: AOA distributions for RSSA = RSSB = RSS

A RSS measurement is the sum of the strength of the desired signal coming from the transmitter and the sum of strengths of interfering signals. The sum of strengths of interfering signals is the noise measurement observed by beams (A) and (B). Insignificant variation in noise was noticed as shown in Figure 4.5; therefore, the noise was ignored to simplify our computations.

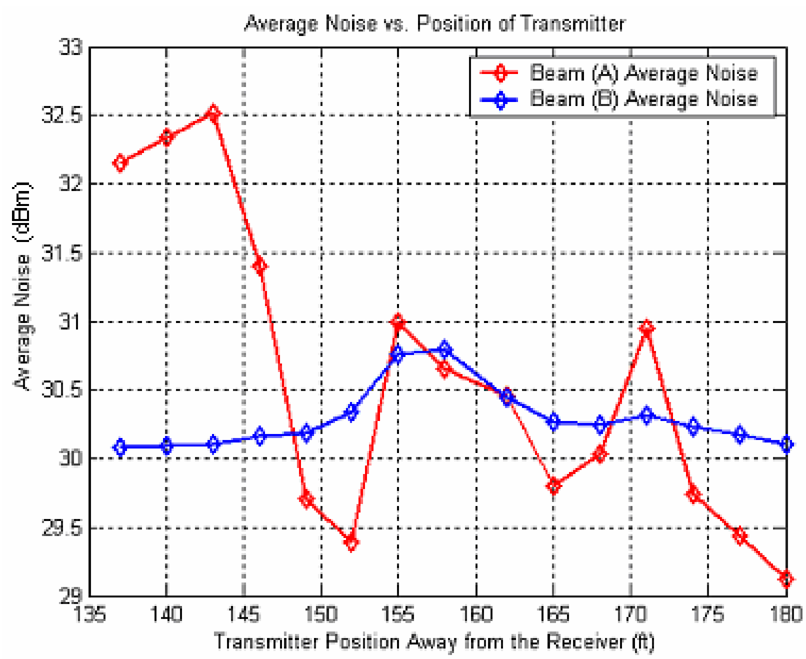


Figure 4.5: The average of the noise observed by the receiver

5. RSS as a Function of Distance and AOA

It is important now to explore the behavior of the received signal strength (RSS) observed by the two directional antennas as a function of distance and AOA. The behavior of RSS provides the motivation to estimate the distance and AOA between the receiver and transmitter. Figure 5.1 shows RSSA and RSSB as a function of distance and AOA. The data used to generate the plot in Figure 5.1 is not the data collected for the purpose of this project as described in section (4). The data used in Figure 5.1 was collected using two Pacific Wireless MD24-12 antennas and they were separated by 60^{o} angle as shown in Figure 5.2 [10]. This preliminary data was collected for the purpose of studying the behavior of RSS as a function of distance and AOA. Two important conclusions can be derived from Figure 5.1:

1. At a constant AOA or θ, the larger the distance between the transmitter and receiver, the smaller the values of RSSA and RSSB:

   $$\mathrm{RSSA}(16\mathrm{m},\theta) > \mathrm{RSSA}(32\mathrm{m},\theta) > \mathrm{RSSA}(47\mathrm{m},\theta) \text{ for any given } \theta \tag{6}$$

   $$\mathrm{RSSB}(16\mathrm{m},\theta) > \mathrm{RSSB}(32\mathrm{m},\theta) > \mathrm{RSSB}(47\mathrm{m},\theta) \text{ for any given } \theta \tag{7}$$

2. At a constant distance, and at $\theta \approx 30^o$, RSSA observes its largest value and it decreases gradually moving away from $\theta = 30^o$. Similarly, at $\theta \approx -30^o$, RSSB observes its largest value and decreases gradually away from $\theta = -30^o$.

Hence, from the above two observations, we choose to model our accumulated RSS measurements to estimate the distance and AOA:

$$d = h(RSSA,RSSB) \tag{8}$$

$$\theta = g(RSSA,RSSB) \tag{9}$$

As an example, if we know that RSSA = RSSB, by looking at Figure 5.1, we can estimate θ, $\theta \approx 5^o$.

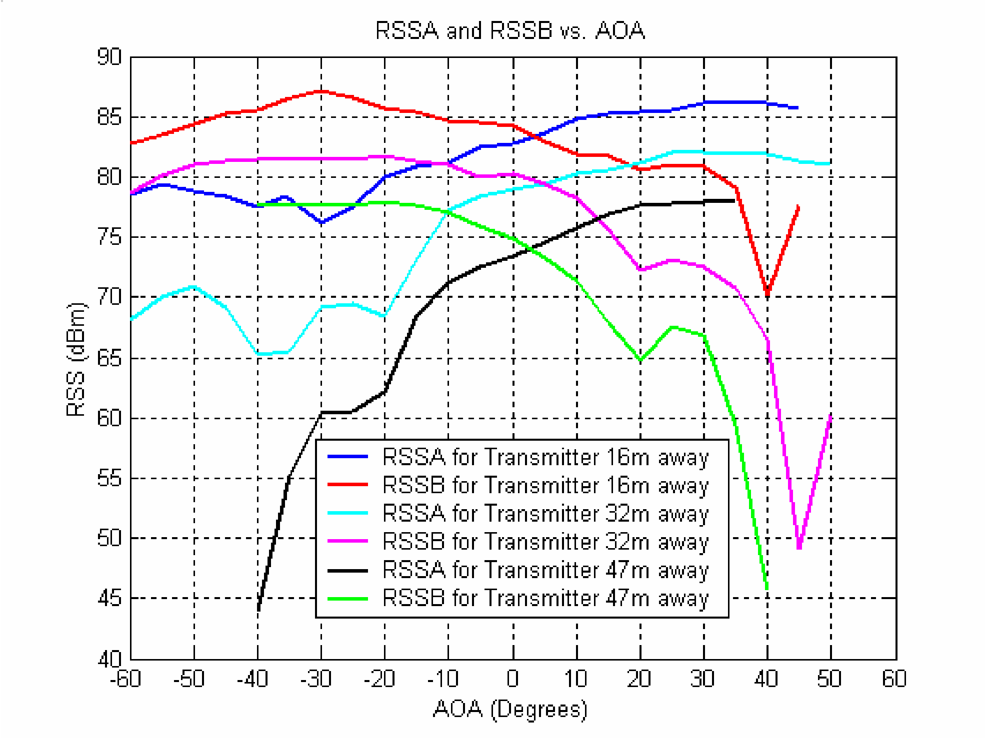


Figure 5.1: RSSA and RSSB vs. Actual AOA

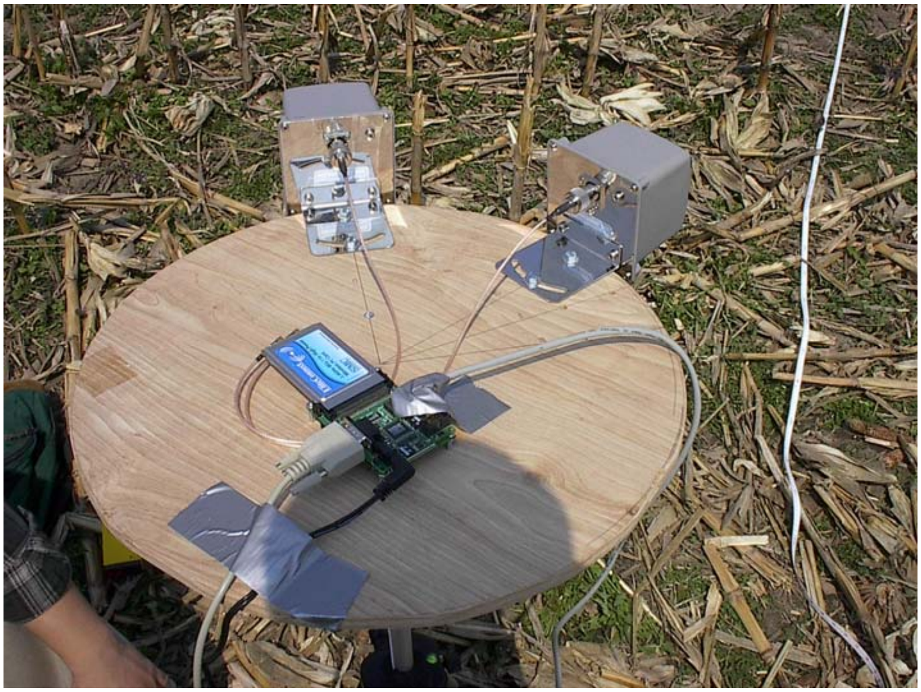


Figure 5.2: The two Pacific Wireless MD24-12 antennas

6. Posterior Probability

Given the collected RSS measurements, the posterior probability $P(d,\theta \mid RSSA, RSSB)$ can be computed. Using Bayes rule, we notice that:

$$P(d,\theta \mid RSSA, RSSB) = \frac{P(RSSA, RSSB \mid d,\theta)P(d,\theta)}{P(RSSA, RSSB)} \tag{10}$$

$$\Rightarrow \frac{P(d,\theta)}{P(RSSA, RSSB)} = \frac{P(d,\theta \mid RSSA, RSSB)}{P(RSSA, RSSB \mid d,\theta)} \tag{11}$$

$$\frac{P(d,\theta \mid RSSA, RSSB)}{P(RSSA, RSSB \mid d,\theta)} = \frac{\dfrac{n(d,\theta \mid RSSA, RSSB)}{n(RSSA, RSSB)}}{\dfrac{n(RSSA, RSSB \mid d,\theta)}{n(d,\theta)}} \tag{12}$$

Where,

- $n(d,\theta \mid RSSA, RSSB)$ = number of packets such that $RSS = \{RSSA, RSSB\}$ and (d,θ) is the transmitter position.
- $n(RSSA, RSSB \mid d,\theta)$ = number of packets such that $RSS = \{RSSA, RSSB\}$ and (d,θ) is the transmitter position.
- $n(RSSA, RSSB)$ = number of packets such that $RSS = \{RSSA, RSSB\}$.
- $n(d,\theta)$ = number of packets such that (d,θ) is the transmitter position.

$$\Rightarrow n(d,\theta \mid RSSA, RSSB) = n(RSSA, RSSB \mid d,\theta) \text{ and } n(d,\theta) = 400 \tag{13}$$

Combining (11), (12) and (13),

$$\frac{P(d,\theta)}{P(RSSA, RSSB)} = \frac{400}{n(RSSA, RSSB)} \tag{14}$$

Finally, substituting (14) into (10),

$$P(d,\theta \mid RSSA, RSSB) = \frac{400 \times P(RSSA, RSSB \mid d,\theta)}{n(RSSA, RSSB)} \tag{15}$$

In (15), the probability $P(RSSA, RSSB \mid d,\theta)$ and $n(RSSA, RSSB)$ are computed from the accumulated packets to obtain the desired probability $P(d,\theta \mid RSSA, RSSB)$.

7. Angle Estimation

To estimate the angle of arrival (AOA), we pursue various computational techniques: Conditional mean estimation, Conditional mean/mode estimation and Conditional median estimation. To test each approach, we estimate the AOA from the observed RSSA and RSSB as the transmitter was positioned on the locations on the curves of 149*ft*, 152*ft*, 155*ft*, 158*ft*, 162*ft*, 165*ft*, and 168*ft* as shown in the yellow area of Figure 7.1, for a total of 91 positions. As shown in Figure 7.1, the distributions used to estimate the distance and angle in the yellow area usually have more samples than distributions used to estimate positions outside the yellow area. Hence, estimating positions inside the yellow area provides a practical and general situation to test our approach. Appendix (A) provides a table with complete results of each method. Next, we discuss each method and finally compare and analyze the results.

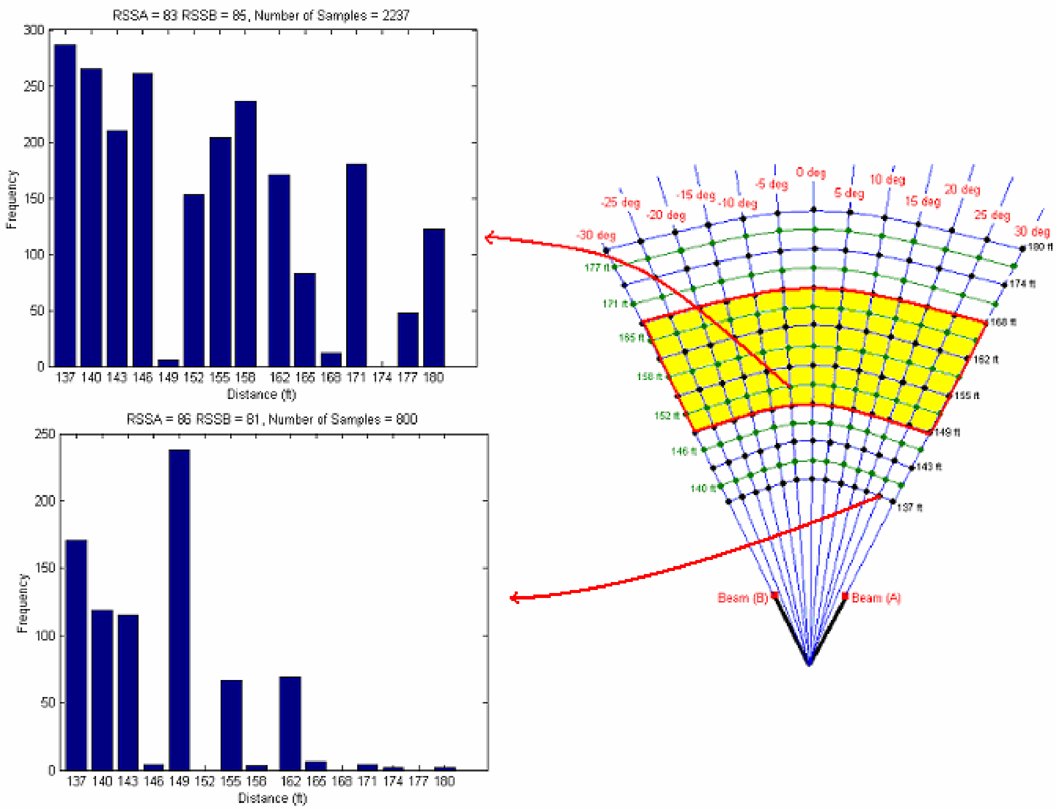


Figure 7.1: The chosen yellow area to test the proposed position estimation technique

7.1 Conditional Mean Estimation

One simple approach to estimate the AOA is to compute the mean of the AOA distribution of the observed RSSA and RSSB. Figure 7.2 shows the mean AOA for all 186 $RSS = \{RSSA, RSSB\}$ cases in scale image representation. Using this approach, we were able to estimate the AOA with 4.6 degrees mean absolute error and 3.4 degrees standard deviation of absolute error.

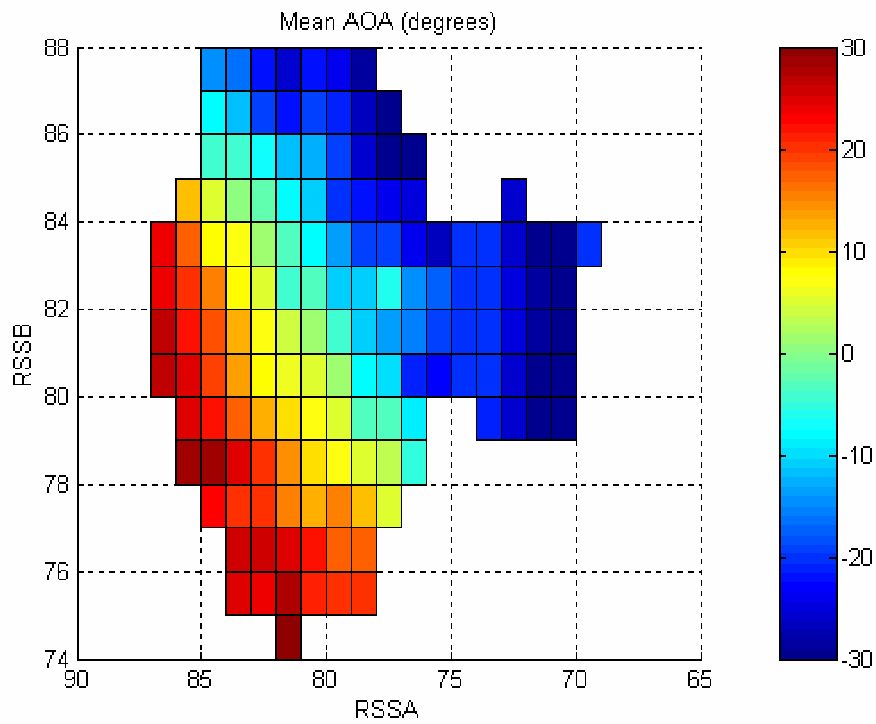


Figure 7.2: Mean AOA vs. {RSSA, RSSB}

7.2 Conditional Mean/Mode Estimation

The mean of the AOA distribution of the observed RSSA and RSSB is not always a good estimator. The AOA distribution in Figure 7.3 is a good example; in this case, the mean value does not accurately represent the distribution because given the available samples, the behavior of the distribution beyond -30^{o} is unknown. Hence, the mode value is a better representation of the distribution in this case. In this approach, we pursue the following guidelines:

1. If the mode value of the distribution is less than -20^{o} or larger than 20^{o}, we choose the mode value to estimate the AOA.
2. Otherwise, if the mode value of the distribution is larger than -20^{o} and less than 20^{o}, we choose the mean value to estimate the AOA.

Using this approach, we were able to estimate the AOA with 4.3 degrees mean absolute error and 3.8 degrees standard deviation of absolute error.

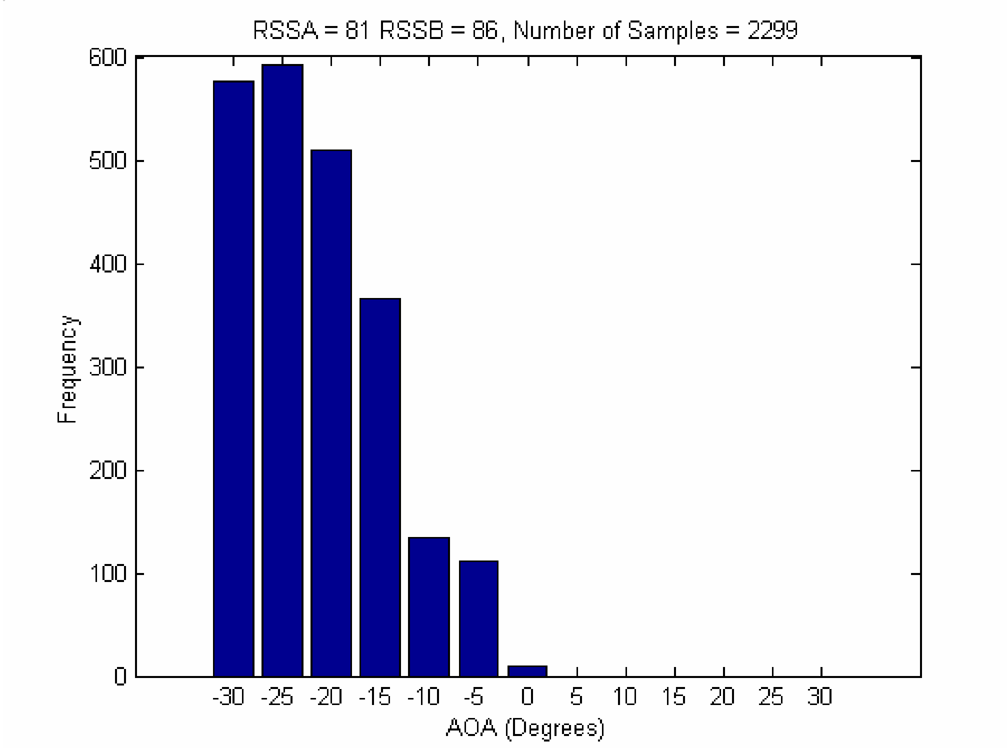


Figure 7.3: A special case where the mode is a better representation of the distribution than the mean.

7.3 Conditional Median Estimation

Another possible approach to estimate the AOA is to compute the median of the AOA distributions. Figure 7.4 shows the median AOA for all 186 $RSS = \{RSSA, RSSB\}$ pairs in scale image representation. Using this approach, we were able to estimate the AOA with 4.5 degrees mean absolute error and 4.1 degrees standard deviation of absolute error.

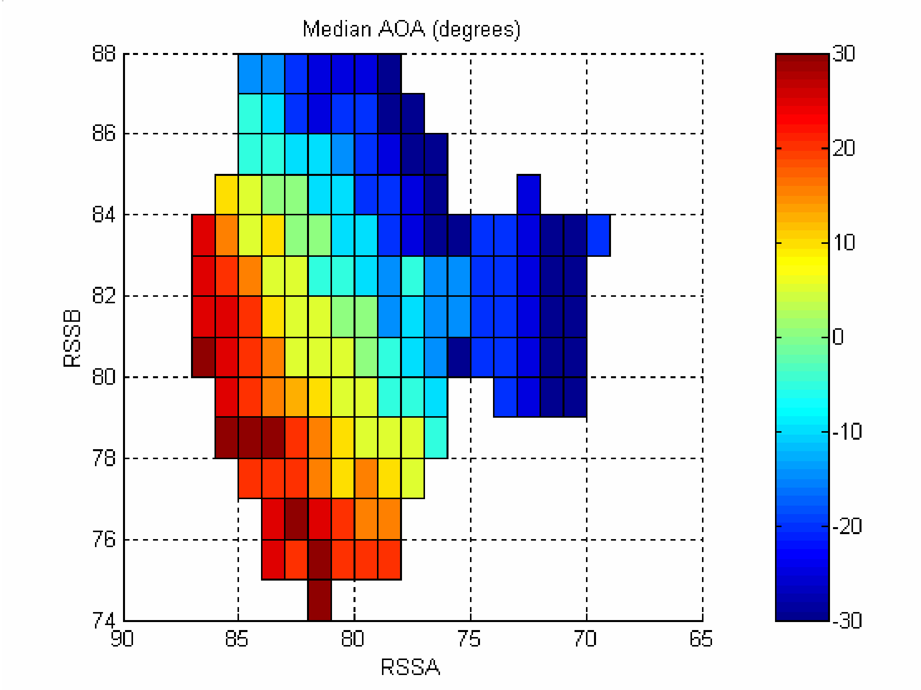


Figure 7.4: Median AOA vs. {RSSA, RSSB}

7.4 Results Analysis

As shown in Table 7.1, the conditional mean/mode estimation provides the best results with the least average error. As expected from estimation theory, the conditional mean estimator minimizes standard deviation, whereas the conditional median estimator minimizes the mean absolute error. Figure 7.5 illustrates that the conditional mean estimator does not perform that well for actual AOA less than -20^{o} or larger than 20^{o}. Similarly, for actual AOA larger than -5^{o} and less than 5^{o}, the conditional median estimator does not perform that well.

Method	Mean Absolute AOA Error (degrees)	Standard Deviation of Mean Absolute AOA Error (degrees)
Mean Estimation	4.6	3.4
Mean/Mode Estimation	4.3	3.8
Median Estimation	4.5	4.1

Table 7.1: AOA estimation results

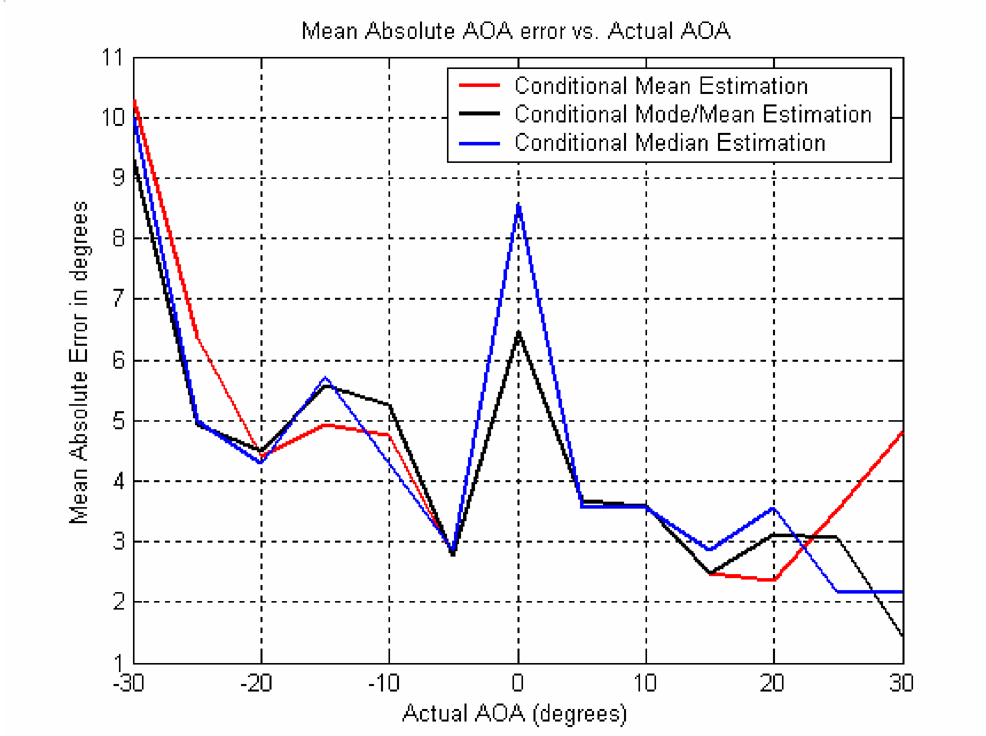


Figure 7.5: AOA mean absolute error vs. actual AOA

8. Distance Estimation

To estimate the distance between the transmitter and receiver, we pursue different computational techniques: Conditional mean estimation using one RSS, Line fit using one RSS, and Conditional mean estimation using the two RSSs observed by the two-antenna array. As we did with AOA estimation, to test each approach, we estimate the distance between the transmitter and receiver as the transmitter was positioned on the same 91 locations mentioned previously. Appendix (B) provides a table with complete results of each method. Next, we discuss each method and finally compare and analyze the results.

8.1 Conditional Mean Estimation Using One RSS

In this method, the distance between the transmitter and receiver is estimated by computing the mean of the distance distribution for the larger value between RSSA and RSSB. As we mentioned previously in section (4), there are nine distance distributions for each $RSSA$, ranging from 79 to 87, observed when beam (A) was pointing directly at the transmitter. Similarly, there are ten distance distributions for each ten $RSSB$ values, ranging from 79 to 88. Implementing this approach, we were able to estimate the distance with 5.8 feet mean absolute error and 3.4 feet standard deviation of absolute error.

8.2 Line Fit Estimation Using One RSS

Figures 8.1 and 8.2 demonstrate that the mean of the distance distributions for RSSA and the distance distributions for RSSB eventually decrease as RSSA and RSSB increases, respectively. Using least-square estimation, we were able to estimate the equations of the best-fit lines shown in Figures 8.1 and 8.2.

$$\hat{d} = -3.78 * RSSA + 471.1 \tag{16}$$

$$\hat{d} = -3.29 * RSSB + 435.5 \tag{17}$$

Equation (16) is used to estimate the distance if RSSA is larger than RSSB. On the other hand, equation (17) is used to estimate the distance if RSSB is larger than RSSA. Implementing this computational technique, we were able to estimate the distance with 8.0 feet mean absolute error and 4.0 feet standard deviation of absolute error.

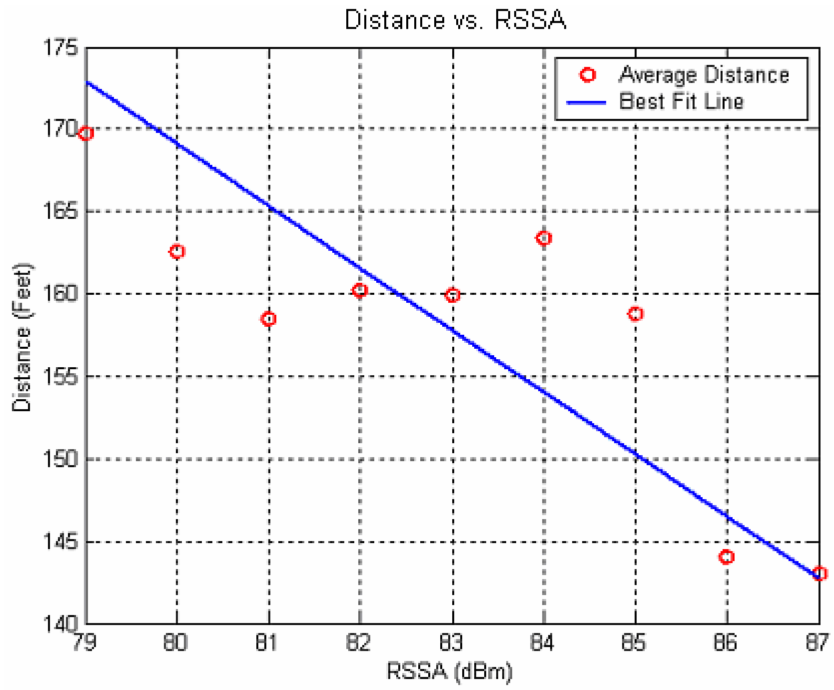


Figure 8.1: The mean of all nine RSSA distance distributions and best-fit line.

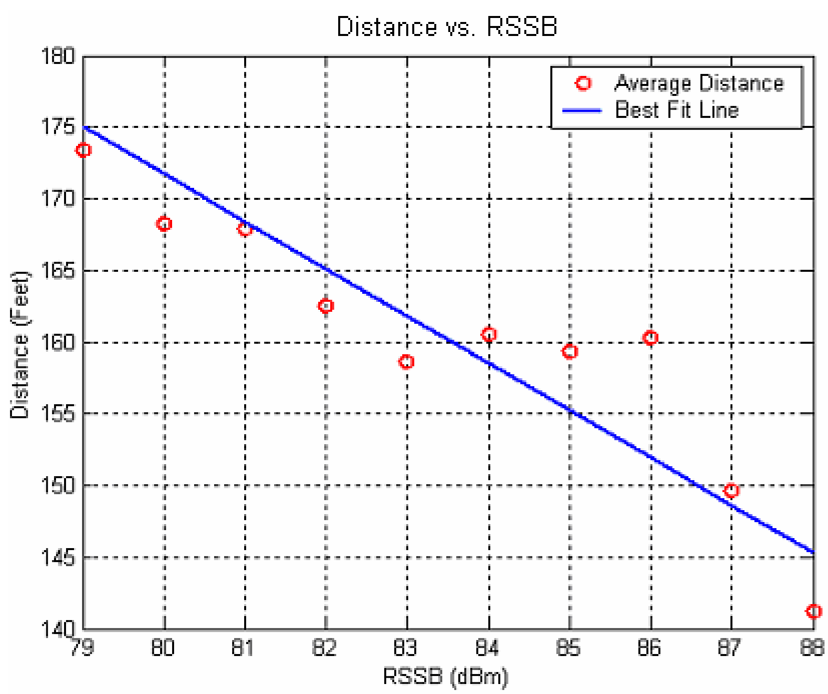


Figure 8.2: The mean of all ten RSSB distance distributions and best-fit line.

8.3 Conditional Mean Estimation Using {RSSA, RSSB}

Another approach to estimate the distance is to compute the mean of the distance distribution of the observed RSSA and RSSB pair. Figure 8.3 shows the mean distance for all 186 $RSS = \{RSSA, RSSB\}$ cases in scale image representation. Using this approach, we were able to estimate the distance with 5.3 feet mean absolute error and 4.2 feet standard deviation of absolute error.

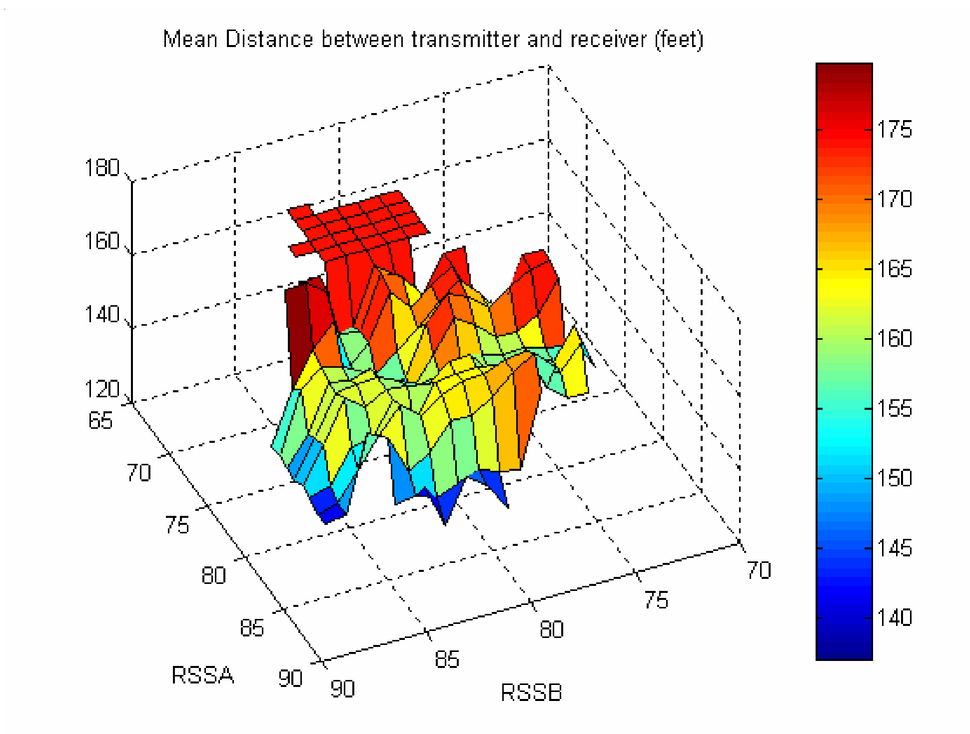


Figure 8.3: 3-D visualization of mean distance vs. {RSSA, RSSB}

8.4 Results Analysis

From Table 8.1, we notice that the conditional mean estimation using two RSSs provides the best results with the least average error. The two RSSs, RSSA and RSSB, observed by beam (A) and beam (B) of the receiver, respectively, provide more information about the position of the transmitter than a single RSS. Hence, by using

RSSA and RSSB, we were able to compute a better estimate of the distance between the receiver and transmitter.

Method	Mean Absolute Distance Error (Feet)	Standard Deviation of Absolute Error (Feet)
Mean Estimator Using one RSS	5.8	3.4
Best Line Fit Using one RSS	8.0	4.0
Mean Estimator Using Two RSSs	5.3	4.2

Table 8.1: Distance estimation results

9. Position Estimation

After estimating the AOA and distance separately, we now combine the results of sections (7) and (8) to obtain the polar coordinates of the transmitter with respect to the receiver. Throughout this computation, we assume that the receiver is located at the origin of the x-y coordinates system. We choose the Conditional Mean AOA Estimation and Conditional Mean/Mode AOA Estimation that provide the best AOA estimation. Similarly, we choose the Conditional Mean Distance Estimation Using One RSS and Conditional Mean Distance Estimation Using {RSSA, RSSB}. Hence, we can pursue four methods of position estimation:

1. Method #1: Conditional Mean AOA Estimation/ Conditional Mean Distance Estimation Using One RSS.
2. Method #2: Conditional Mean AOA Estimation/ Conditional Mean Distance Estimation Using {RSSA, RSSB}.
3. Method #3: Conditional Mean/Mode AOA Estimation/ Conditional Mean Distance Estimation Using One RSS.
4. Method #4: Conditional Mean/Mode AOA Estimation/ Conditional Mean Distance Estimation Using {RSSA, RSSB}.

As we did in sections (7) and (8), to test each approach, we estimate the position of the transmitter as the transmitter was positioned on the same 91 locations mentioned previously. Appendix (C) provides a table with complete results of each method. Next, we discuss each method.

9.1 Method #1

Figure 9.1 demonstrates that the estimated positions are confined within very small areas because the distance is estimated using only 19 possible distance distributions. Hence, we do not have enough distance distributions or information to obtain more accurate estimations. Also, we notice that there are only few estimated positions in the area of actual AOA less than -20^{o} or larger than 20^{o} because the conditional mean AOA estimator does not provide good estimation in those areas. By using this method, we were able to estimate positions with 15.1 feet mean absolute distance error between actual and estimated positions and 8.5 feet standard deviation of absolute error.

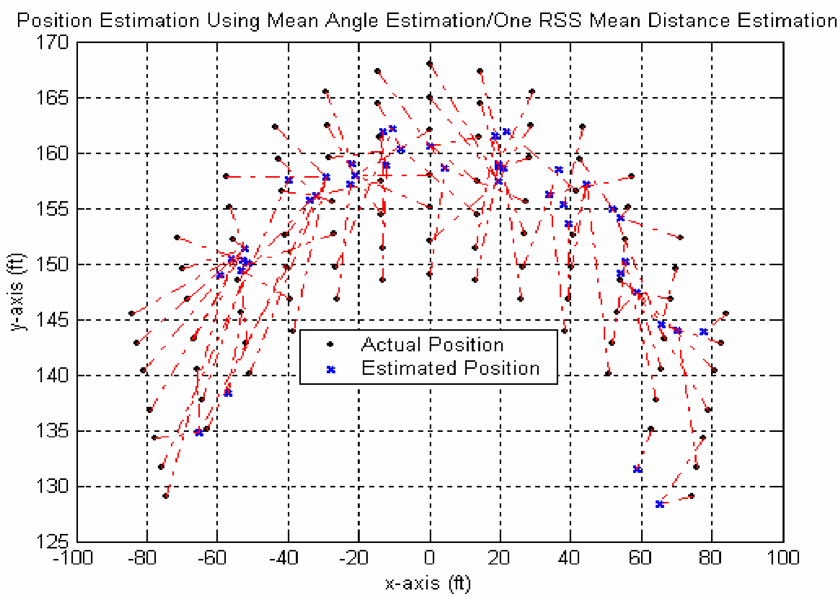


Figure 9.1: Estimated positions using method #1

9.2 Method #2

Figure 9.2 shows that the estimated positions are more spread compared to the estimated positions in Figure 9.1. The reason is that the conditional mean distance estimation using two RSSs {RSSA, RSSB} utilizes 186 distance distributions to estimate the distance. But we still notice that there are only few estimated positions in the area of actual AOA less than -20^{o} or larger than 20^{o} because of the limitations of the conditional mean AOA estimator in these areas. Implementing this method, we were able to estimate positions with 14.8 feet mean absolute distance error between actual and estimated positions and 9.1 feet standard deviation of absolute error.

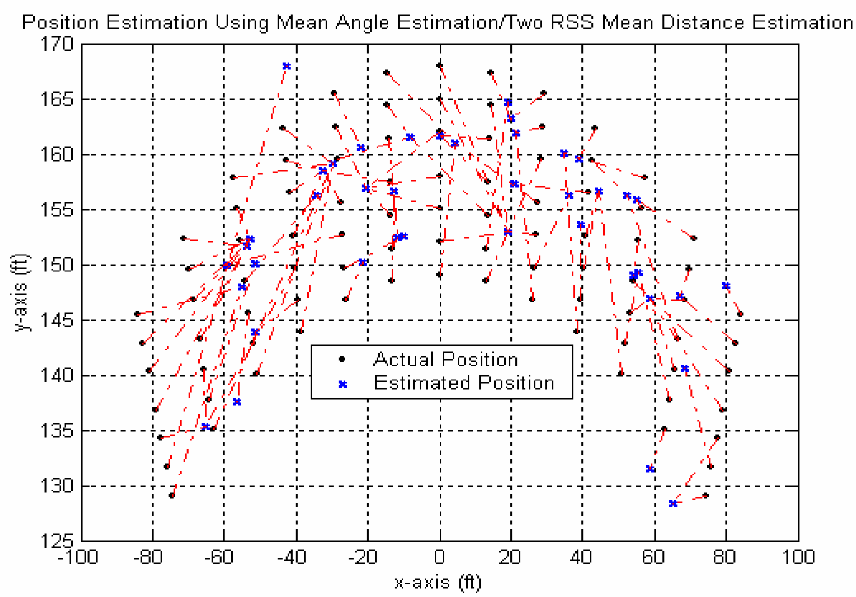


Figure 9.2: Estimated positions using method #2

9.3 Method #3

Figure 9.3 illustrates that the estimated positions are confined in small areas again because of the limitations of the conditional mean distance estimation using one RSS observation. However, there is a relatively (compared to Figures 9.1 and 9.2) large number of estimated positions in the area of actual AOA less than -20^o or larger than 20^o because of the strengths of the conditional mean/mode AOA estimator in these areas. Using this method, we were able to estimate positions with 14.9 feet mean absolute distance error and 9.2 feet standard deviation of absolute error.

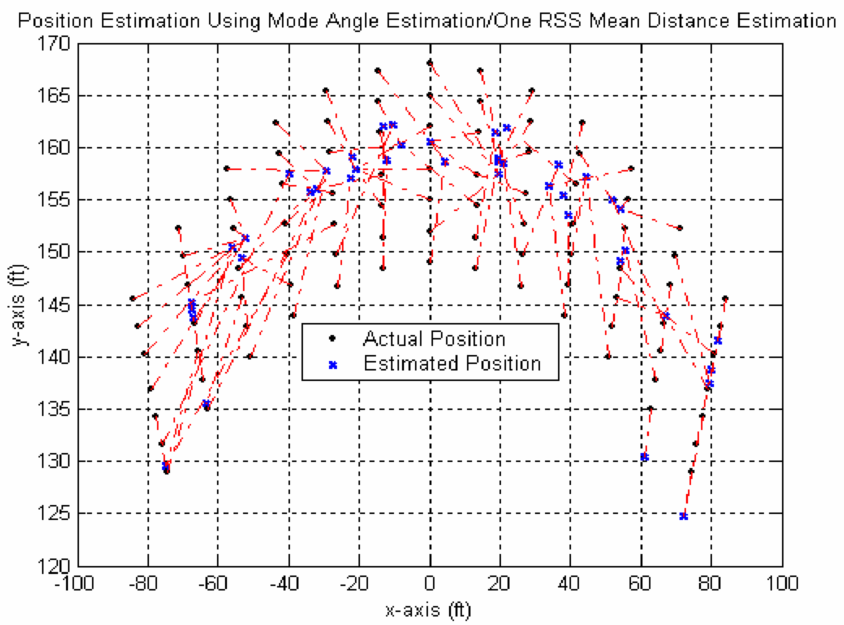


Figure 9.3: Estimated positions using method #3

9.4 Method #4

Figure 9.4 shows the strengths of the conditional mean/mode AOA estimator and the conditional mean distance estimator using two RSS observations. The estimated positions are relatively more spread compared to Figures 9.1, 9.2 and 9.3. Also, implementing this method provided the least mean absolute distance error between actual and estimated positions of 14.5 feet and standard deviation of 9.8 feet.

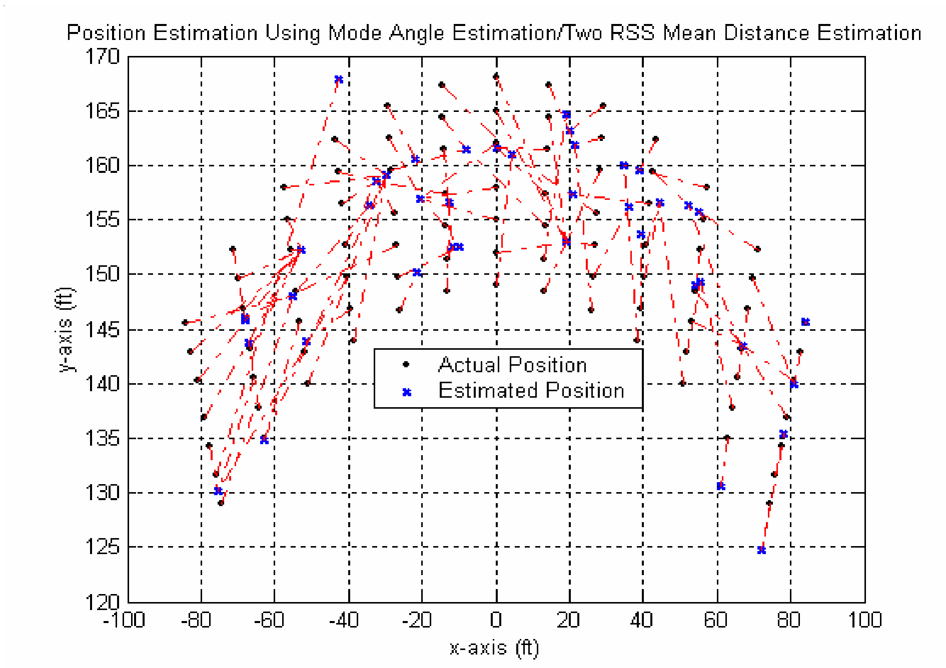


Figure 9.4: Estimated positions using method #4

Finally, Table 9.1 shows the mean absolute distance error and standard deviation for all four methods.

Method	Mean Absolute Distance Error between Actual and Estimated Position (Feet)	Standard Deviation of Absolute Error between Actual and Estimated Position (Feet)
Method #1	15.1	8.6
Method #2	14.8	9.1
Method #3	14.9	9.2
Method #4	14.5	9.8

Table 9.1: Position estimation results

10. Conclusion

In this paper, we provided a practical experiment to estimate a node's position by utilizing the AOA sensing capabilities of the beacon node. We built a signal-map to estimate the position of the transmitter from the observed {RSSA, RSSB}. We found that by utilizing the AOA sensing capability of one beacon node, we can uniquely (one solution) estimate the position of unknown neighboring node. We also found that by using a multiple-antenna array receiver, more information about the position of the transmitter can be obtained; hence more accurate distance estimation can be deduced. We were able to estimate a node's position with 14.5*ft* mean absolute error and ratio of average error area over surveyed area of about 0.0925 as shown in Figure 10.1.

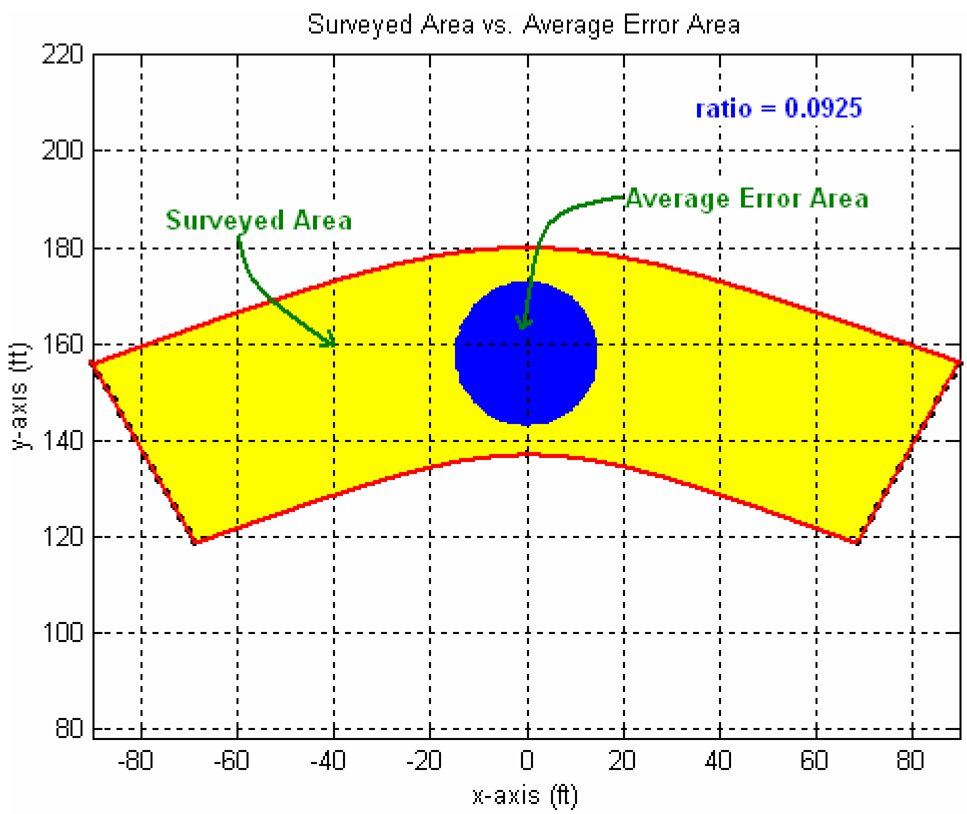


Figure 10.1: Surveyed area vs. Average error area.

11. References

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Appendix A: Angle Estimation Results

The three methods pursued to estimate the AOA are:

1. Method #1: Conditional Mean Estimator
2. Method #2: Conditional Mean/Mode Estimator
3. Method #3: Conditional Median Estimator

Actual r (Feet)	Actual θ (Degrees)	Method #1 θ Estimate	Method #2 θ Estimate	Method #3 θ Estimate
149	-30	-20.34	-20.34	-20.00
149	-25	-10.56	-10.56	-10.00
149	-20	-10.56	-10.56	-10.00
149	-15	-10.56	-10.56	-10.00
149	-10	-8.03	-8.03	-10.00
149	-5	-4.61	-4.61	-5.00
149	0	1.55	1.55	0
149	5	7.05	7.05	10.00
149	10	7.54	7.54	5.00
149	15	12.25	12.25	10.00
149	20	15.83	15.83	15.00
149	25	24.06	25.00	25.00
149	30	27.00	30.00	30.00
152	-30	-14.21	-14.21	-15.00
152	-25	-18.91	-25.00	-25.00
152	-20	-19.68	-20.00	-20.00
152	-15	-19.68	-20.00	-20.00
152	-10	-3.71	-3.71	-5.00
152	-5	-4.41	-4.41	-5.00
152	0	7.05	7.05	10.00
152	5	6.58	6.58	5.00
152	10	12.99	12.99	10.00
152	15	14.31	14.31	15.00
152	20	20.37	20.00	20.00
152	25	20.37	20.00	20.00
152	30	25.97	30.00	30.00
155	-30	-25.77	-30.00	-25.00
155	-25	-25.77	-30.00	-25.00
155	-20	-22.34	-25.00	-25.00
155	-15	-25.77	-30.00	-25.00
155	-10	-21.61	-25.00	-25.00
155	-5	-7.49	-7.49	-10.00
155	0	-7.49	-7.49	-10.00

155	5	-2.81	-2.81	0
155	10	7.05	7.05	10.00
155	15	15.83	15.83	15.00
155	20	21.79	25.00	25.00
155	25	21.79	25.00	25.00
155	30	27.00	30.00	30.00
158	-30	-19.13	-20.00	-20.00
158	-25	-19.13	-20.00	-20.00
158	-20	-12.33	-12.33	-15.00
158	-15	-11.61	-11.61	-10.00
158	-10	-11.61	-11.61	-10.00
158	-5	-11.61	-11.61	-10.00
158	0	-7.49	-7.49	-10.00
158	5	0.14	0.14	0
158	10	1.55	1.55	0
158	15	15.83	15.83	15.00
158	20	21.79	25.00	25.00
158	25	18.50	18.50	20.00
158	30	21.79	25.00	25.00
162	-30	-19.13	-20.00	-20.00
162	-25	-19.13	-20.00	-20.00
162	-20	-19.13	-20.00	-20.00
162	-15	-11.61	-11.61	-10.00
162	-10	-2.81	-2.81	0
162	-5	-4.41	-4.41	-5.00
162	0	-7.49	-7.49	-10.00
162	5	0.14	0.14	0
162	10	7.05	7.05	10.00
162	15	7.64	7.64	5.00
162	20	20.00	20.00	20.00
162	25	21.79	25.00	25.00
162	30	21.79	25.00	25.00
165	-30	-19.13	-20.00	-20.00
165	-25	-19.13	-20.00	-20.00
165	-20	-19.13	-20.00	-20.00
165	-15	-11.61	-11.61	-10.00
165	-10	-7.49	-7.49	-10.00
165	-5	-2.81	-2.81	0
165	0	7.05	7.05	10.00
165	5	7.05	7.05	10.00
165	10	7.54	7.54	5.00
165	15	18.50	18.50	20.00
165	20	19.44	20.00	20.00
165	25	24.46	30.00	25.00
165	30	24.46	30.00	25.00

168	-30	-20.34	-20.34	-20.00
168	-25	-19.42	-25.00	-20.00
168	-20	-10.56	-10.56	-10.00
168	-15	-10.56	-10.56	-10.00
168	-10	-7.82	-7.82	-10.00
168	-5	1.55	1.55	0
168	0	7.05	7.05	10.00
168	5	7.54	7.54	5.00
168	10	7.04	7.04	5.00
168	15	13.72	13.72	15.00
168	20	12.25	12.25	10.00
168	25	19.44	20.00	20.00
168	30	28.31	30.00	30.00

Appendix B: Distance Estimation Results

The three methods pursued to estimate the distance are:

1. Method #1: Conditional mean estimator using one RSS
2. Method #2: Line fit using one RSS
3. Method #3: Conditional mean estimator using {RSSA, RSSB}

Actual *r* (Feet)	Actual θ (Degrees)	Method #1 *r* Estimate	Method #2 *r* Estimate	Method #3 *r* Estimate
149	-30	160.5295	158.5295	157.7737
149	-25	160.5295	158.5295	161.9014
149	-20	160.5295	158.5295	161.9014
149	-15	160.5295	158.5295	161.9014
149	-10	158.6704	161.8266	151.7239
149	-5	162.4857	165.1237	157.1779
149	0	158.6704	161.8266	161.0174
149	5	159.9462	157.7975	154.0649
149	10	159.9462	157.7975	163.2831
149	15	159.9462	157.7975	163.7495
149	20	163.3977	154.0222	162.7950
149	25	144.0064	146.4716	144.0425
149	30	144.0064	146.4716	144.1086
152	-30	162.4857	165.1237	173.2281
152	-25	158.6704	161.8266	158.6000
152	-20	158.6704	161.8266	152.8723
152	-15	158.6704	161.8266	152.8723
152	-10	162.4857	165.1237	152.8838
152	-5	159.3581	155.2324	152.9392
152	0	158.6704	161.8266	154.0649
152	5	162.5852	169.1234	165.7500
152	10	162.5852	169.1234	160.3243
152	15	158.4966	165.3481	158.5855
152	20	160.2116	161.5728	159.2822
152	25	160.2116	161.5728	159.2822
152	30	160.2116	161.5728	156.4000
155	-30	149.6474	148.6382	150.2451
155	-25	149.6474	148.6382	150.2451
155	-20	149.6474	148.6382	148.7203
155	-15	149.6474	148.6382	150.2451
155	-10	160.2662	151.9353	161.2706
155	-5	159.3581	155.2324	158.2532

155	0	159.3581	155.2324	158.2532
155	5	160.5295	158.5295	161.6889
155	10	159.9462	157.7975	154.0649
155	15	163.3977	154.0222	162.7950
155	20	158.7669	150.2469	158.2239
155	25	158.7669	150.2469	158.2239
155	30	144.0064	146.4716	144.1086
158	-30	160.2662	151.9353	161.1678
158	-25	160.2662	151.9353	161.1678
158	-20	159.3581	155.2324	159.9995
158	-15	159.3581	155.2324	161.7747
158	-10	159.3581	155.2324	161.7747
158	-5	159.3581	155.2324	161.7747
158	0	159.3581	155.2324	158.2532
158	5	160.5295	158.5295	161.5854
158	10	158.6704	161.8266	161.0174
158	15	163.3977	154.0222	162.7950
158	20	158.7669	150.2469	158.2239
158	25	163.3977	154.0222	164.8059
158	30	158.7669	150.2469	158.2239
162	-30	160.2662	151.9353	161.1678
162	-25	160.2662	151.9353	161.1678
162	-20	160.2662	151.9353	161.1678
162	-15	159.3581	155.2324	161.7747
162	-10	160.5295	158.5295	161.6889
162	-5	159.3581	155.2324	152.9392
162	0	159.3581	155.2324	158.2532
162	5	160.5295	158.5295	161.5854
162	10	159.9462	157.7975	154.0649
162	15	163.3977	154.0222	158.7518
162	20	158.7669	150.2469	158.5380
162	25	158.7669	150.2469	158.2239
162	30	158.7669	150.2469	158.2239
165	-30	160.2662	151.9353	161.1678
165	-25	160.2662	151.9353	161.1678
165	-20	160.2662	151.9353	161.1678
165	-15	159.3581	155.2324	161.7747
165	-10	159.3581	155.2324	158.2532
165	-5	160.5295	158.5295	161.6889
165	0	158.6704	161.8266	154.0649
165	5	159.9462	157.7975	154.0649
165	10	159.9462	157.7975	163.2831
165	15	163.3977	154.0222	164.8059
165	20	163.3977	154.0222	165.2189
165	25	158.7669	150.2469	161.6913

165	30	158.7669	150.2469	161.6913
168	-30	160.5295	158.5295	157.7737
168	-25	159.3581	155.2324	160.8080
168	-20	160.5295	158.5295	161.9014
168	-15	160.5295	158.5295	161.9014
168	-10	160.5295	158.5295	162.1395
168	-5	158.6704	161.8266	161.0174
168	0	158.6704	161.8266	154.0649
168	5	159.9462	157.7975	163.2831
168	10	160.2116	161.5728	164.4830
168	15	159.9462	157.7975	164.1797
168	20	159.9462	157.7975	163.7495
168	25	163.3977	154.0222	165.2189
168	30	163.3977	154.0222	168.1753

Appendix C: Position Estimation Results

The four methods pursued to estimate the position:

1. Method #1: Conditional Mean AOA Estimation/ Conditional Mean Distance Estimation Using One RSS.
2. Method #2: Conditional Mean AOA Estimation/ Conditional Mean Distance Estimation Using {RSSA, RSSB}.
3. Method #3: Conditional Mean/Mode AOA Estimation/ Conditional Mean Distance Estimation Using One RSS.
4. Method #4: Conditional Mean/Mode AOA Estimation/ Conditional Mean Distance Estimation Using {RSSA, RSSB}.

Actual *r* (Feet)	Actual θ (Degrees)	Method #1 Absolute Position Error (Feet)	Method #2 Absolute Position Error (Feet)	Method #3 Absolute Position Error (Feet)	Method #4 Absolute Position Error (Feet)
149	-30	28.4788	27.2663	28.4788	27.2663
149	-25	40.5421	41.1106	40.5421	41.1106
149	-20	27.9361	28.6265	27.9361	28.6265
149	-15	16.6234	17.6375	16.6234	17.6375
149	-10	11.0176	5.8371	11.0176	5.8371
149	-5	13.5276	8.2445	13.5276	8.2445
149	0	10.5298	12.7293	10.5298	12.7293
149	5	12.2561	7.4115	12.2561	7.4115
149	10	12.7951	15.7739	12.7951	15.7739
149	15	13.2170	16.5446	13.2170	16.5446
149	20	18.3341	17.8514	18.3341	17.8514
149	25	5.5434	5.5110	4.9936	4.9575
149	30	9.1454	9.0923	4.9936	4.8914
152	-30	44.4269	49.3725	44.4269	49.3725
152	-25	17.7987	17.7691	6.6704	6.6000
152	-20	6.7262	1.2173	6.7262	1.2173
152	-15	14.3309	12.4805	14.3309	12.4805
152	-10	20.1732	16.7403	20.1732	16.7403
152	-5	7.5294	1.8246	7.5294	1.8246
152	0	20.2188	18.9207	20.2188	18.9207
152	5	11.4357	14.4276	11.4357	14.4276
152	10	13.3856	11.6400	13.3856	11.6400

152	15	6.7607	6.8464	6.7607	6.8464
152	20	8.2718	7.3497	8.2718	7.3497
152	25	15.0549	14.5371	15.0549	14.5371
152	30	13.7134	11.7097	8.2116	4.4000
155	-30	12.4509	12.2265	5.3526	4.7549
155	-25	5.7305	5.1783	14.3241	14.1367
155	-20	8.2156	8.8340	14.3241	14.6585
155	-15	29.0826	29.0349	40.1170	40.1204
155	-10	32.3188	32.5954	41.4804	41.7471
155	-5	8.0999	7.5416	8.0999	7.5416
155	0	20.9861	20.7144	20.9861	20.7144
155	5	22.1898	22.5807	22.1898	22.5807
155	10	9.5046	8.0203	9.5046	8.0203
155	15	8.7091	8.1283	8.7091	8.1283
155	20	6.1736	5.8511	14.1943	14.0371
155	25	9.5698	9.3552	3.7669	3.2239
155	30	13.4880	13.4064	10.9936	10.8914
158	-30	30.2270	30.3922	30.2270	30.3922
158	-25	16.4503	16.6434	16.4503	16.6434
158	-20	21.2628	21.3559	21.2628	21.3559
158	-15	9.4775	10.1765	9.4775	10.1765
158	-10	4.6680	5.8733	4.6680	5.8733
158	-5	18.3536	18.8238	18.3536	18.8238
158	0	20.7707	20.6559	20.7707	20.6559
158	5	13.7522	14.0279	13.7522	14.0279
158	10	23.3324	23.6876	23.3324	23.6876
158	15	5.8792	5.3293	5.8792	5.3293
158	20	4.9974	4.9349	13.8384	13.7953
158	25	19.0051	19.5254	19.0051	19.5254
158	30	22.6981	22.6475	13.8384	13.7953
162	-30	30.5703	30.6182	30.5703	30.6182
162	-25	16.5893	16.5657	16.5893	16.5657
162	-20	2.9968	2.5886	2.9968	2.5886
162	-15	9.8583	9.5721	9.8583	9.5721
162	-10	20.2720	20.2939	20.2720	20.2939
162	-5	3.1140	9.2036	3.1140	9.2036
162	0	21.1526	21.2471	21.1526	21.2471
162	5	13.7664	13.7388	13.7664	13.7388
162	10	8.5479	11.3702	8.5479	11.3702
162	15	20.9390	20.8477	20.9390	20.8477
162	20	3.2331	3.4620	3.2331	3.4620
162	25	9.5571	9.7400	3.2331	3.7761
162	30	23.1970	23.2401	14.3596	14.4684
165	-30	31.1641	31.1258	31.1641	31.1258
165	-25	17.3103	17.1314	17.3103	17.1314

165	-20	5.3380	4.5613	5.3380	4.5613
165	-15	11.1224	10.1820	11.1224	10.1820
165	-10	9.0727	9.7802	9.0727	9.7802
165	-5	7.6561	7.0621	7.6561	7.0621
165	0	20.8693	22.4401	20.8693	22.4401
165	5	7.6941	12.3287	7.6941	12.3287
165	10	8.6110	7.2505	8.6110	7.2505
165	15	10.1517	10.0694	10.1517	10.0694
165	20	2.2778	1.6426	2.2778	1.6426
165	25	6.4179	3.6508	15.4345	14.6284
165	30	16.8430	16.1336	6.2331	3.3087
168	-30	28.6425	29.2581	28.6425	29.2581
168	-25	18.1273	17.5485	8.6419	7.1920
168	-20	28.0334	27.8118	28.0334	27.8118
168	-15	14.7480	14.1517	14.7480	14.1517
168	-10	9.7374	8.5876	9.7374	8.5876
168	-5	20.8642	20.0545	20.8642	20.0545
168	0	22.1290	24.1899	22.1290	24.1899
168	5	10.8490	8.7286	10.8490	8.7286
168	10	11.5029	9.2703	11.5029	9.2703
168	15	8.8438	5.3195	8.8438	5.3195
168	20	23.5729	22.8157	23.5729	22.8157
168	25	16.7313	16.4126	16.7313	16.4126
168	30	6.7093	4.9559	4.6023	0.1753